THE PROBLEM OF THE TRAJECTORIES OF CIRCULAR GAS JETS IN A NONUNIFORM SIDE STREAM

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We describe an engineering method for constructing the trajectories of circular gas jets in a side drift stream with an arbitrary velocity field along the jet.

For many engineering problems in the construction of the trajectories of circular gas jets propagating in side drift streams the velocity field of the drift flow is nonuniform along the direction of the outflow of the jet (for example, the propagation of gas jets from the gas turbine engine of a helicopter, in the inductive air flows from the lifting rotor).

The equations of the trajectories of gas jets in a drift flow, the velocity distribution in which along the jet is subject to a law of the form $w = f(y)$, where w is a continuously differentiable function, can be obtained analytically by the same methods as for a uniform drift flow. But in some cases it is difficult to use analytical methods due to the complexity of the velocity field of the drift flow. The engineering method proposed in this article makes it possible to construct the trajectories of circular jets in a side drift stream with an arbitrary flow velocity distribution along the jet.

The essence of the method is as follows. We compare the trajectories of the jets propagating in a uniform drift flow with the auxiliary curves constructed for the same conditions (σ_{02}/q_{01} and α_0) by summing the momentum vectors of a jet and the drift flow and so determine the functional form of the correction coefficient $\xi = f(q_{02}/q_{01};\ y/d_0)$. This coefficient is obtained as the ratio of the ordinate x/d_0 of the trajectory to the ordinate x^*/d_0 of the auxiliary curve at corresponding sections of the jet $y/d_0 = \text{const}$.

Then for the nonuniform drift flow we construct the auxiliary curves by the same method as for a uniform flow, after which the ordinates x/d_0 of points on the unknown trajectory are determined by multiplying the ordinates of the auxiliary curves x^*/d_0 by the appropriate coefficient ξ.

In this method we can take any equation which is suitable for the appropriate range of ratios q_{02}/q_{01} and given α_0 as the initial equation for constructing the trajectories of the circular jets. But the best results are obtained by using the empirical equations of the trajectories. As for the theoretical equations of the trajectories of the circular gas jets in a side drift stream, most of them contain the empirical coefficient C_n, which is used as the average aerodynamic drag coefficient of the jet. This coefficient is not constant; it depends on the interactions between the jet and the stream: the angle of outflow of the jet in the stream, the ratio of the velocity heads at the initial cross section of the jet and in the stream, the structure of the stream at the initial cross section. To illustrate this, Fig. 1 shows C_n as a function of the above factors for the theoretical equation of the trajectories obtained by Abramovich's method [1]. To a large extent, C_n depends also on the extent of the trajectory of the jet used in its definition, where the local aerodynamic drag coefficients at various cross sections of the jet may differ from each other significantly from one section to another.

All this makes it complicated to use the theoretical equations as the initial equations for determining the trajectories of jets in a nonuniform side stream in the method considered below.

Of the empirical equations in practice it is more convenient to use those in which the effect of the angle α_0 on the trajectory is taken into account by a separate term. In this case we first use the method to construct a trajectory in a nonuniform flow for $\alpha_0 = 90°$ and then determine the required trajectory, the effect of α_0 being taken into account in the same way as in the initial equation for a uniform drift flow.

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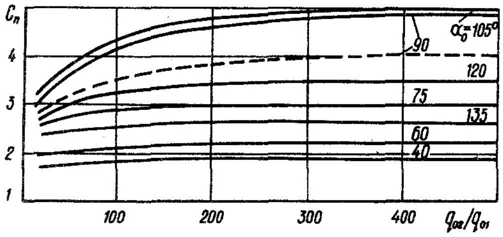

Fig. 1

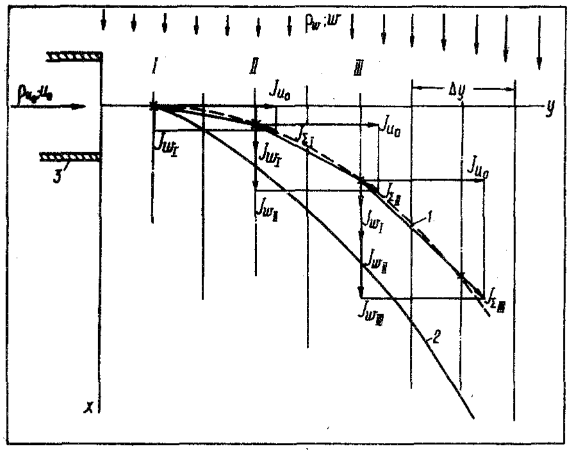

Fig. 2

Fig. 1. Graphs of $C_n = f(q_{02}/q_{01})$ for $a = 0.066$ (continuous lines) and $a = 0.060$ (dotted line).

Fig. 2. Construction of trajectories by the method of auxiliary curves: 1) auxiliary curve; 2) trajectory; 3) nozzle.

The auxiliary curves for $\alpha_0 = 90°$ are constructed in the following sequence:

1) the jet is divided by parallel planes perpendicular to its initial direction into separate segments. The distance between neighboring planes is Δy (Fig. 2);

2) the momentum flux of the drift flow in any of the segments of the jet between two planes is computed from the equation

$$J_{w_i} = 2\int_0^{\Delta y} q_{01} h \, dy' \ldots ; \tag{1}$$

3) the initial momentum flux of the jet, which is subsequently taken as constant in magnitude and direction for all sections of the jet, is calculated as

$$J_{u_0} = 2q_{02} F_0 \ldots ; \tag{2}$$

4) then we calculate successively at each of the segments the geometrical sum of the vectors $\bar{J}_{u_0}$ and $\sum_0^i \bar{J}_{w_i}$, as shown in Fig. 2. In the first segment the vectors are measured from a point on the jet axis in the middle of the segment. For the other segments the origins of the vectors for the geometrical addition are the points of intersection of the mean lines of the segments with the direction of the summation momentum vector $\bar{J}_\Sigma$ from the preceding segments;

5) we construct the auxiliary curves as the lines passing through the initial points of the summation vectors in each segment.

In constructing the auxiliary curves it is convenient in practice for the subsequent integration of (1) to take a linear equation for the jet width h:

$$h = h_s + ny'. \tag{3}$$

We also take $\Delta y = d_0$ for convenience when ρ_w and n are constant and thus reduce (1) to the form

$$J_{wi} = \rho_w d_0^2 \left[h_s \int_0^1 w^2 d\left(\frac{y'}{d_0}\right) + n \int_0^1 w^2 \frac{y'}{d_0} \, d\left(\frac{y'}{d_0}\right) \right] \ldots , \tag{4}$$

where h_s is calculated from the equation

$$h_s = h_0 + ny_s. \tag{5}$$

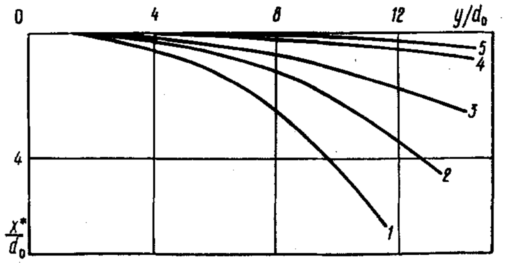

Fig. 3

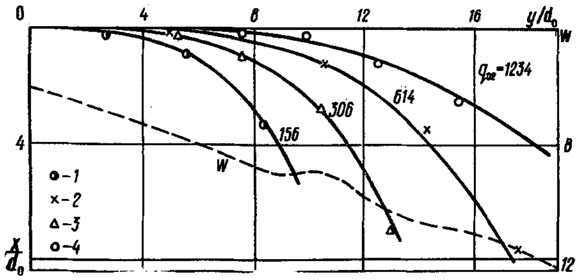

Fig. 4

Fig. 3. The auxiliary curves for a uniform side drift flow with $\alpha_0 = 90°$: 1) $q_{02}/q_{01} = 25$; 2) 50; 3) 100; 4) 250; 5) 500.

Fig. 4. Trajectories of jets in a nonuniform drift flow with $\alpha_0 = 90°$ (W, m/sec). Continuous lines) method of auxiliary curves; dotted line) velocity field of drift flow; 1-4) experimental points: 1) $q_{02} = 156$; 2) 614; 3) 306; 4) 1234.

We consider in some detail the method of auxiliary curves, using for an example the initial equation for the trajectories in a uniform drift flow the equation of Ivanov [2], which can be written as

$$\frac{x}{d_0} = 195\ a^2 \left(\frac{q_{01}}{q_{02}}\right)^{1.3} \left(\frac{y}{d_0}\right)^3 + \frac{y}{d_0}\ \mathrm{ctg}\,\alpha_0. \tag{6}$$

Figure 3 shows the auxiliary curves constructed by the method of summing the momentum vectors for the case of a uniform drift flow when $\alpha_0 = 90°$ and $q_{02}/q_{01} = 25$-500. In the calculations we took $h_0 = d_0$ and $n = 0.2$ in Eqs. (4) and (5). By analyzing the family of auxiliary curves we obtain the following equation in logarithmic coordinates:

$$\frac{x^*}{d_0} = 0.5 \left(\frac{q_{01}}{q_{02}}\right) \left(\frac{y}{d_0}\right)^{2.3}. \tag{7}$$

We now write the equation for the ordinates of the jet trajectories in terms of those of the auxiliary curves as follows:

$$\frac{x}{d_0} = \frac{x^*}{d_0}\ \xi \ldots, \tag{8}$$

from which ξ, by (6) and (7) for $\alpha_0 = 90°$, is given by

$$\xi = 390\ a^2 \left(\frac{q_{01}}{q_{02}}\right)^{0.3} \left(\frac{y}{d_0}\right)^{0.7} \ldots \tag{9}$$

Then the required trajectories for the circular jets in a nonuniform drift flow are constructed in the following order:

1) By the method of summing the momentum vectors we construct the auxiliary curves. We use the same values $h_0 = d_0$ and $n = 0.2$ as for a uniform drift flow.

2) From Eq. (8) we determine the ordinates of the required trajectories of the jets at various sections Δy.

In determining ξ for each section Δy from (9), we take q_{01} as the average velocity head of the drift flow for that section.

3) If $\alpha_0 \neq 90°$ in this case, the ordinates of the trajectory in the nonuniform drift flow are determined from the equation

$$\frac{x}{d_0} = \frac{x^*}{d_0}\ \xi + \frac{y}{d_0}\ \mathrm{ctg}\,\alpha_0 \ldots, \tag{10}$$

which, as distinct from (8), has a second term of the same form as in the initial equation of the trajectory (6).

To check the method described above the author constructed suitable experiments. In these the gas jets were released into the open working section of a wind tunnel from a nozzle of diameter 14 mm at the outlet cross section. The coefficient of turbulent structure a for a free jet from this nozzle was 0.06. A system of directional blades and grids in the wind tunnel circuit made it possible to obtain velocity fields in the working section which were uniform along the initial direction of the gas jet.

Figure 4 compares the trajectories constructed by the method of auxiliary curves, for $\alpha_0 = 90°$, with the corresponding experimental data.

NOTATION

x is the coordinate in the direction of the drift flow;
y is the coordinate in the direction of the outflow of the jet from the nozzle;
y' is the distance along the y-axis from the origin of the segment;
α is the angle between the jet axis and the direction of the drift flow;
h is the jet width;
F is the cross-sectional area;
d is the cross-sectional diameter;
n is the angular coefficient of jet expansion;
u is the jet velocity;
w is the drift flow velocity;
q_{02} is the velocity head at initial jet cross section;
q_{01} is the velocity head of drift flow;
ρ is the density;
J is the momentum;
a is the coefficient of jet structure;
C_n is the aerodynamic drag coefficient;
ξ is the correction coefficient.

Subscripts

0 denotes the initial jet cross section;
s denotes the start of segment;
u denotes the jet;
w denotes the drift flow;
i denotes the current value;
* denotes the auxiliary curve.

LITERATURE CITED

1. G. N. Abramovich, The Theory of Turbulent Jets [in Russian], Fizmatgiz (1960).
2. Yu. V. Ivanov, Author's Abstract of Candidate's Dissertation [in Russian], TsKII, Leningrad (1950).